

WEDNESDAY 8:30AM.

TEXT BOOKS

⇒ K.A. STRAUD Engineering mathematics 6th edition
⇒ Higher Engineering mathematics G.O. Chandrasekhar

ABE 302

COURSE OUTLINE

Probability and Statistics.

Probability space theorem, Conditional probability and Independent variable, random Variables, discrete and Continuous distribution, mean and Variance, binomials, Poisson and hyper geometry. Centre limit theorem, Statistical inference

Elementary Sample theory for normal population or mean population & Variance

Chi-square test, Simple linear regression and engineering application

Statistics, the exploration and analysis of data by Box, Jenkins & Joy DEVOBE.

~~Schaum series~~ ⇒ Schaum Series, ⇒ Yold tube - video on Statistical Inferences

Electromechanical ~~EEE 346 (8-10)~~ Digital elec ~~EEE 321 (10-12)~~ Instrumentation ~~EEE 312 (10-12)~~

DAY ~~EEE 302 (8-10)~~, ~~EEE 352 (10-12)~~ ⇒ power by Sanni

ABE 302 (8-10), EEE 392 (practical 10-12).

Control ~~EEE 324 (7-10) digital~~ ~~ABE 302 (10-12)~~ ~~Communic~~ ~~EEE 332 (12-3)~~

Control EEE 302 (9:00 to ---)

MON 8pm ~~11:00~~ (10 press forward from fresh) 91.5 ~~11:00~~ (ghankogbi) (9:00 pm)

91.5 (13 press forward from fresh)

Tues. 91.5 (11:00 pm)

Wed Fresh 7:30-8:30 (lowuio eranna)

6 press forward after fresh is Adaba (88.9)

7 press " " " " is Voice (89.9)

MON 7:30 Voice 7 press forward from fresh

PROBABILITY

5-5-2021

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Probability is the measure of likelihood

that a particular event will occur in any one trial / experiment carried out in a prescribed condition

NOTATION

The probability that a certain event A will occur is denoted by $P(A) = p(A)$ and also equal to (P Success).

Success / Failure

When an event occur in any one trial is called Success but when it fails to occur it's called failure.

In N trials there are x Success, there will also be (N-x) failure.

$$\frac{x}{N} = P(\text{Success})$$

$$\frac{N-x}{N} = P(\text{Failure})$$

It's applied that $P(A) + P(\text{Not } A) = 1$

$$\text{i.e. } \frac{x}{N} + \frac{N-x}{N} = 1 \quad \text{i.e. } P(A) + P(\bar{A}) = 1$$

Event "not A" is complement of event A and often written as $\bar{A}$

TYPES OF PROBABILITY

- Empirical or experimental probability.
- Classical or Theoretical probability.

EMPIRICAL PROBABILITY

This is form from what we know as experiment.

EXPECTATION ⇒ This is defined as the

number of trials and probability that event A will occur.

$$E = N \times P(A)$$

$$\text{Where } P(A) = \frac{1}{12}$$

CLASSICAL PROBABILITY

The classical approach is based on theoretical number of ways ^{which} are possible

for an event to occur. Based on assumption

Classical probability $\text{Prob}(P)$ of an event (A) occurring is defined by;

$$P(A) = \frac{\text{Numbers of ways in which event will occur}}{\text{Total number of all possible outcomes.}}$$

MUTUALLY EXCLUSIVE AND NON-MUTUALLY EXCLUSIVE EVENT

Mutually exclusive events are events which cannot occur together. Single event ^(note, it can occur but not same time)

Mutually non-exclusive events are events which occur together simultaneously when there is a pair of events e.g. Scoring 2's in a pair of dice.

ADDITIONAL LAWS OF PROBABILITY

For a given events (A) and (B) are mutually exclusive if A and B are mutually exclusive the probability is either event (A) or (B) but not both.

$$P(A \text{ or } B) = P(A) + P(B) = \frac{x+y}{n} = \frac{x}{n} + \frac{y}{n}$$

$$\text{If events are non-exclusive, } P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

INDEPENDENT EVENT AND DEPENDENT EVENT

Events are Independent when the occurrence of event doesn't affect the probability of the occurrence of the second event (Replacement).

Events are said to be dependent when one event does affect the probability of the occurrence of the second event (non-replacement).

CONDITIONAL PROBABILITY.

Conditional probability of an event B

occurring when given an event A have already

taken place vice versa. In case of an event

stated above, $P\left(\frac{B}{A}\right)$ is the probability

LIBRARY READING

Heading Additional laws

Exp. mutually non-exclusive.

In rolling a die, the proba of scoring a multiple of 3.
 $(3, 6) = \frac{2}{6} = P(A)$

In rolling a multiple of 3.
 $(2, 4, 6) = \frac{3}{6} = P(B)$

The find prob of A or B occurring.
Sol.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

To find $P(A \text{ and } B) = ?$

Note $\Rightarrow$ prob of scoring multiple of 3 or 2
is $P(A) + P(B) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

Note at prob of multiple of 3 = 3, 6

" " " " 2 = 2, 4, 6

One outcome (6) appears in both set of outcomes. $\therefore$ The outcome has been counted twice.

The $P(\text{six}) = \frac{1}{6} = P(A \text{ and } B)$ (i.e. picking 1)

$$\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \\ = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{2}{3}$$

OR

OR

multiple of 3 = 3, 6
multiple of 2 = 2, 4, 6

i.e. 5 elements

Since 6 exists twice, $\therefore$ count 6 once and not the twice.

i.e. 3, 6, 2, 4 or 3, 2, 4, 6.

(Counting the element, they are 4 elements)

Also, Since a fair die is 6 sides.

$$\therefore \text{prob of } (A \text{ or } B) = \frac{4 \text{ element}}{6} \\ = \frac{2}{3}$$

IDENTIFICATION OF EXCLUSIVE AND NON EXCLUSIVE

EXCLUSIVES:

$$\Rightarrow A = \{\text{heart}\} \quad B = \{\text{ace of spades}\}$$

$$\Rightarrow A = \{5 \text{ of clubs}\} \quad B = \{10 \text{ of clubs}\}$$

$$\Rightarrow A = \{\text{Card} < 10\} \quad B = \{\text{King of diamond}\}$$

NON-EXCLUSIVES

$$\Rightarrow A = \{\text{ace}\} \quad B = \{\text{Black card}\}$$

$$\Rightarrow A = \{\text{red card}\} \quad B = \{\text{Jack}\}$$

EXAMPLE:

$\Rightarrow$ A Single Card is drawn from a pack of 52 playing cards.

Q. If event $A = \{\text{drawing an ace}\}$ and event $B = \{\text{drawing a seven}\}$. The prob of drawing either ace or seven i.e.

$$P(A \text{ or } B) = ?$$

Sol.

Note $\text{ace} = 13$

Sol

Box these are mutually exclusive events
in any one trial.

$$\therefore P(A \text{ or } B) = P(A) + P(B).$$

4 Aces in the pack $\therefore P(A) = P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$

4 Sevens in the pack $\therefore P(B) = P(\text{Seven}) = \frac{4}{52} = \frac{1}{13}$

$$\therefore P(A \text{ or } B) = P(\text{Ace or Seven}) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

2) If event $A = \{ \text{drawing a King} \}$ and event

$B = \{ \text{drawing a red card} \}$, then $P(A \text{ or } B) = ?$

Sol

These are non-exclusive events since we could draw the King of hearts or the King of diamonds.

$$\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

4 Kings in the pack $\therefore P(A) = P(\text{King}) = \frac{4}{52} = \frac{1}{13}$

26 red cards in the pack $\therefore P(B) = P(\text{red card}) = \frac{26}{52} = \frac{1}{2}$

2 red Kings in the pack $\therefore P(A \text{ and } B) = P(\text{red King}) = \frac{2}{52} = \frac{1}{26}$

$$\therefore P(A \text{ or } B) = \frac{1}{13} + \frac{1}{2} - \frac{1}{26} = \frac{2+13-1}{26}$$

$$= \frac{14}{26} = \frac{7}{13}$$

! for mutually exclusive $P(A \text{ or } B) = P(A) + P(B)$

for mutually non-exclusive $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

THINGS TO NOTE.

$\Rightarrow$ Mutually non-exclusive event $\Rightarrow$ They are events that happen simultaneously. e.g. in rolling a die, the event of obtaining a multiple of 3 & the event of obtaining a multiple of two can occur together to give 6 i.e. it occurs together if a 6 is thrown.

THINGS TO NOTE.

$\Rightarrow$ DIE $\Rightarrow$ This is a small cube. ~~dot~~ dots mark on their faces. plural of the die is dice. On throwing a die, the outcome is one of dots on its upper face.

$\Rightarrow$ CARDS $\Rightarrow$ A pack of Cards consists of four suits i.e. Spades, Hearts, Diamonds and clubs. Each suit consists of 13 cards, nine cards numbered 2, 3, 4, ..., 10, an ACE, a KING, a QUEEN and a JACK or KNAVE. Colour of spades and clubs is black and that of hearts and diamonds is red. Kings, Queens and Jacks are known as face card.

$\Rightarrow$ EXHAUSTIVE EVENTS OR SAMPLE SPACE $\Rightarrow$ The set of all possible outcomes of a single performance of an experiment is exhaustive events or sample space. Each outcome is called a sample point. In case of tossing a coin once, $S = \{H, T\}$ is the sample space. Two outcomes - head and tail - constitute an exhaustive event box no other outcome is possible. (in throwing a die, there are 6 exhaustive cases) In throwing a die, there are exhaustive cases of 6.

$\Rightarrow$ RANDOM EXPERIMENT $\Rightarrow$ There are experiments in which results may be altogether different even though they are performed under identical conditions. They are known as random experiments. Tossing a coin or throwing a die is random experiment. (if a die is thrown continuously many not by dice, then the die is not a random experiment, getting 1, 2, 3, 4, 5, 6 is an event)

$\Rightarrow$ TRIAL AND EVENT $\Rightarrow$ Performing a random experiment is called a TRIAL and outcome is termed as EVENT. Tossing of a coin is a trial and the turning up of head or tail is an event.

$\Rightarrow$ EQUALLY LIKELY EVENTS $\Rightarrow$ Two events are said to be "equally likely" if one of them can't be expected in preference to the other. For instance, if we draw a card from a well-shuffled pack, we may get any card, then the 52 different cases are equally likely.

$\Rightarrow$ INDEPENDENT EVENT $\Rightarrow$ Two events may be independent, when the actual happening of one doesn't influence in any way the probability of the happening of the other.

e.g. The event of getting head on first coin and an event of getting tail on the second coin in a simultaneous throw of two coins are independent.

$\Rightarrow$ Mutually exclusive event (incompatible) Two events are known as mutually exclusive when the occurrence of one of them excludes the occurrence of the other. For example, on tossing of a coin, either we get head or tail, but not both. In rolling a die, the event of getting a 6 and the event of getting a 1 are mutually exclusive.

• Compound Event $\Rightarrow$ When two or more events occur in composition with each other, the simultaneous occurrence is called a compound event. When a die is thrown, getting a 5 or 6 is a compound event.

Favourable Event $\Rightarrow$ The events, which ensure the required happening, are said to be favourable events. E.g. in throwing a die, to get the even numbers, 2, 4, 6 are favourable cases. While the favourable cases of getting an odd number are 1, 3, 5. (1,3,5) (2,4,6) (3,3)

• Conditional probability $\Rightarrow$ The prob of happening an event A such that event B has already happened, is called the conditional probability of happening of A on the condition that B has already happened. It is usually denoted by $P(A/B)$.

$\Rightarrow$ Odds in favour of an event and odds against an event.

If number of favourable ways = m ,

Number of not-favourable events = n

Odds in favour of the event = $\frac{m}{n}$

Odds against the event = $\frac{n}{m}$

$\Rightarrow$ Classical definition of probability $\Rightarrow$ If there are N equally likely, mutually, exclusive, and exhaustive of events of an experiment and n of these are favourable, then the prob of the happening of the event is defined as $\frac{n}{N}$.

$\Rightarrow$ Expected value $\Rightarrow$ If $P_1, P_2, P_3, \dots, P_n$ are the prob of the events $x_1, x_2, x_3, \dots, x_n$ respectively then the expected value

$$E(x) = P_1 x_1 + P_2 x_2 + P_3 x_3 + \dots + P_n x_n$$

$$\therefore E(x) = \sum_{i=1}^n P_i x_i$$

Examples.

$\Rightarrow$ A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the prob that they will both be white?

Sol.

Total no. of balls = $7 + 6 + 5 = 18$.

Out of 18 balls, 2 can be drawn in ${}^{18}C_2$ ways.

Single trial (time)

• The exhaustive no. of cases = ${}^{18}C_2 = \frac{18 \times 17}{2 \times 1} = 153$
out of 7 white balls, 2 can be drawn in 7C_2
 ${}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$ ways
 $\therefore$ Favourable no. of cases = 21
Prob = $\frac{21}{153} = \frac{7}{51}$.

• Four cards are drawn from a pack of cards. Find the prob that:
(i) all are diamonds
(ii) there is one card of each suit
(iii) There are two spades and two hearts.

Sol.

4 cards can be drawn from a pack of 52 cards in;
 ${}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1} = 270725$

(i) There are 13 diamonds in the pack and 4 can be drawn out of them in ${}^{13}C_4$ ways.
 $\therefore$ favourable number of cases;

$${}^{13}C_4 = \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = 715$$

$$\text{Required Prob} = \frac{715}{270725} = \frac{11}{4169}$$

(ii) Note we have 4 suits.

But here we have two spades and two hearts = 2 spade + 2 heart = 4 suits.

Note each of them has 13 cards.

$$\therefore \text{favourable no. of cases} = {}^{13}C_2 \times {}^{13}C_2$$

$$= \frac{13}{1} \times \frac{13}{1} \times \frac{13}{1} \times \frac{13}{1} = 169 \times 169$$

$$\text{Required Prob} = \frac{169 \times 169}{270725} = \frac{2197}{20825}$$

(iii) There are 4 suits each containing 13 cards.

$$\therefore \text{favourable no. of cases} = {}^{13}C_2 \times {}^{13}C_2$$

$$\text{Required Prob} = \frac{13 \times 13 \times 13 \times 13}{270725} = \frac{2197}{20825}$$

(iv) 2 spades out of 13 can be drawn in ${}^{13}C_2$ ways

also 2 hearts out of 13 can be drawn in ${}^{13}C_2$ ways.

$$\therefore \text{favourable cases} = {}^{13}C_2 \times {}^{13}C_2 = \frac{13 \times 12}{2 \times 1} \times \frac{13 \times 12}{2 \times 1}$$

$$= 78 \times 78$$

$$\text{Required Prob} = \frac{78 \times 78}{270725} = \frac{468}{20825}$$

⇒ A bag contains 50 tickets numbered 1, 2, 3, ..., 50, of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). What is the prob that $x_3 = 30$?

Sol

Exhaustive ~~case~~ no. of cases: ${}^{50}C_5$

If $x_3 = 30$, the 2 tickets with no x_1 and x_2 must come out of 29 tickets numbered 1 to 29, and this can be done in ${}^{29}C_2$ ways. The other two tickets with numbers x_4 and x_5 must come out of the 20 tickets numbered 31 to 50 and this can be done in ${}^{20}C_2$ ways.

∴ Favourable number of cases = ${}^{29}C_2 \times {}^{20}C_2$

$$\text{Required prob} = \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$$

$$= \frac{\frac{29 \times 28}{2 \times 1} \times \frac{20 \times 19}{2 \times 1}}{\frac{50 \times 49 \times 48 \times 47 \times 46}{5 \times 4 \times 3 \times 2 \times 1}} = \frac{551}{15134}$$

19-5-2021

CLASS

BINOMIAL DISTRIBUTION.

Binomial formulae.

$$b(x; n, p) = {}^nC_x \cdot p^x \cdot (1-p)^{n-x}$$

where;

x = Total no. of successes

p = probability of a success in an individual trial

n = number of trials

~~DEF~~

$${}^nC_x = \frac{n!}{(n-x)!}$$

$$P = {}^nC_r \cdot p^r \cdot q^{n-r}$$

⇒ A coin is tossed 10 times, what is the prob. of getting exactly 6x

Sol

$p = 0.5$, — Success

failure $q = 1 - 0.5 = 0.5$ i.e. $q = 1 - p$

$n = 10$

Prob of getting 6x = 6.

$$\begin{aligned} P_6 &= {}^nC_x \cdot p^x \cdot q^{n-x} \\ &= \frac{10!}{6! \times 4!} \times (0.5)^6 \times (0.5)^4 \\ &= {}^{10}C_6 \times p^6 \times (1-p)^{10-6} \end{aligned}$$

$$= \frac{10!}{(10-6)!} \times 0.5^6 \times 0.5^4$$

$$= \frac{10!}{4!} \times 10 \times 9 \times 8$$

POISSON DISTRIBUTION
This occurs when in real life

$$\text{Mean} = np = 10 \times 0.5 = 5$$

$$\text{Variance} = npq = 10 \times 0.5 \times 0.5$$

$$\text{Standard deviation} = \sqrt{npq}$$

POISSON DISTRIBUTION.

$$P(X=N) = \frac{(e^{-\mu})(\mu^x)}{x!}$$

S.D

μ = mean

$\bar{X}$ = means of sample.

$$P(X; \mu) = \frac{(e^{-\mu})(\mu^x)}{x!}$$

where

x = actual number of Successes

that result from experiment.

$$e = 2.71828$$

μ = mean of probability.

properties.

$$\text{mean} = \mu$$

$$\text{Variance} = \mu$$

$$\text{Standard deviation} = \sqrt{\mu}$$

Example

⇒ Average no. of homes sold by Solomon Company is ^{2 homes} 2 homes per day. What is the prob that exactly 3 homes will be sold tomorrow.

Sol.

$\mu = 2$ homes, since 2 homes sold per day

$x = 3$, $e = ?$

$$P(X; \mu) = \frac{(e^{-\mu})(\mu^x)}{x!}$$

$$= \frac{e^{-2} \times 2^3}{3!}$$

HYPERGEOMETRIC

When we take a sample of size n from a population of size capital letter N containing small k items of 9 successes and $N-k$ failures then

If a population comprised N contains small k items of 9 successes and $N-k$ failures then the prob of d hypergeometric random variable X is n , the no. of successes in a random sample of size n is

$$P(X=k) = \frac{\binom{k}{k} \binom{N-k}{n-k}}{\binom{N}{n}}$$

$$= \frac{\binom{k}{k} \binom{N-k}{n-k}}{\binom{N}{n}}$$

where K = Success from population

k = Success from Sample.

EXAMPLE

A deck of cards contains 20 cards. 6 red cards and 14 black cards. 5 cards are drawn random without replacement. What is the prob that exact 4 red cards are drawn.

Sol.

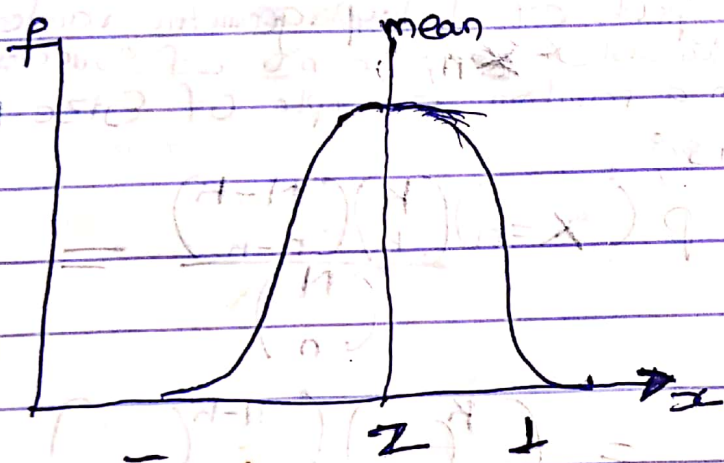
$$\text{Exactly } (k) = 4, K = 6$$

$$N = 20, N - K = 20 - 6 = 14$$

$$n = 5, n - k = 5 - 4 = 1$$

$$\frac{\binom{6}{4} \binom{14}{1}}{\binom{20}{5}} = \frac{\binom{6}{4} \binom{14}{1}}{{}^{20}C_5}$$

Normal Distribution



$Z = \text{Size}$

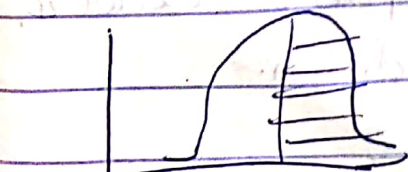
$\bar{X} = \text{mean of Sample}$

$\mu = \text{mean of population}$

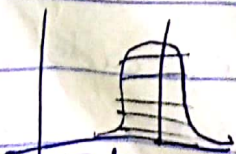
$\sigma = \text{Standard deviation of population}$

$n = \text{Size of Sample}$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



Tail is substrate
 0.5



Tail is
don't subtract
anything.

TEXTBOOK $\Rightarrow$ Advanced Engineering Mathematics
by S. CHAND
H. K. DAS P 781

Reading

BINOMIAL DISTRIBUTION

$$P_{cr} = {}^nC_r p^r q^{n-r}$$

$n = \text{no of trials}$

$r = \text{Total no of Success trials}$

$p = \text{probability of happening of an event}$ ^{in one trial} ~~(Success)~~

$q = \text{probability of not happening of an event in one trial}$

degree of freedom (d.f) = n-1

CLASS

CHI SQUARE (χ^2)

$$\chi^2 = \frac{(O-E)^2}{E} \text{ OR } \sum \frac{(O-E)^2}{E}$$

Example

Class	O	E
ABC	5	5
CVE	6	5
EEE	7	5

$$\chi^2 = \frac{(5-5)^2}{5} + \frac{(6-5)^2}{5} + \frac{(7-5)^2}{5}$$

$$\chi^2 = 0 + 0.2 + 0.8 = 1.0$$

degree of freedom (d.f) = n-1

Table Value < $\chi^2 \rightarrow$ No Significant

" " > " $\rightarrow$ Significant

degree for rows, n=3

for column, n=2

$$\therefore \text{degree of freedom} = (n-1)_{\text{rows}} (n-1)_{\text{column}}$$

$$= (3-1)(2-1)$$

$$= (2)(1) = 2$$

Note Suppose to be given

Note for a complete gn, we suppose to be given a particular percentage of level usually 5% or 0.05 then u use degree of freedom and dis 5% to look a particular number of chi square table.

If Table Value gotten < χ^2 , it is not significant

But if " " > χ^2 , it is significant
i.e. at significance, the theorem is accepted.

Grouped

Observed

Small medium big add

White	10	12	8	30
Red	5	13	10	28
Black	2	5	12	19
add	17	30	30	77

(E) expected as EXPECTED = 77

	Small	medium	large
White	$\frac{30 \times 17}{77}$	$\frac{30 \times 30}{77}$	$\frac{30 \times 30}{77}$
Red	$\frac{17 \times 28}{77}$	$\frac{30 \times 28}{77}$	$\frac{28 \times 30}{77}$
black	$\frac{19 \times 17}{77}$	$\frac{19 \times 30}{77}$	$\frac{19 \times 30}{77}$

i.e.

Small medium large

white	6.6 = 7	11.7 = 12	11.7 = 12
Red	6.2 = 6	11	11
Black	7.1 = 7	7	7

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

from small

$$\chi^2 = \frac{(10-7)^2}{7} + \frac{(12-12)^2}{12} + \frac{(8-12)^2}{12}$$

$$\text{degree of freedom} = (3-1)(3-1) = 4$$

Where n=3 i.e. 3 rows & 3 column

n=3 for rows

n=3 for column

$$\chi^2 = \left(\frac{10-7}{7} \right)^2 + \left(\frac{12-12}{12} \right)^2 + \left(\frac{8-12}{12} \right)^2 + \left(\frac{5-6}{6} \right)^2 + \left(\frac{13-11}{11} \right)^2 + \left(\frac{10-11}{11} \right)^2 + \left(\frac{2-4}{4} \right)^2 + \left(\frac{5-7}{7} \right)^2 + \left(\frac{12-7}{7} \right)^2$$

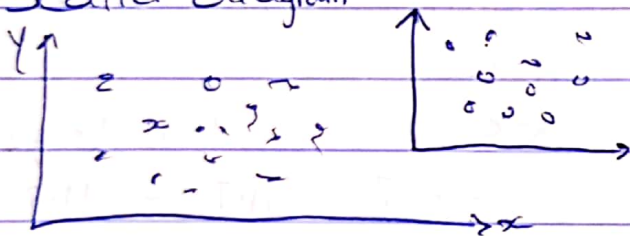
MEASURE OF ASSOCIATION

measure of

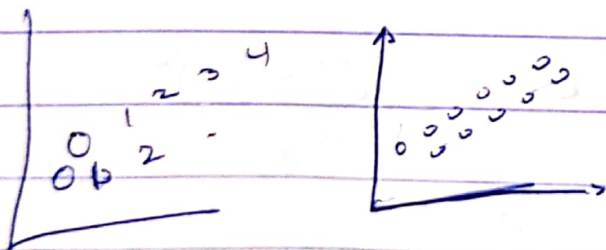
The Study of ^{data} association has to do with bivariate whereby two or more variables are considered as a means of viewing the relationship existing b/w dem. When an increase in one variable leads to an increase in another variable we have a positive association of relationship. However when an increase in a variable leads to decrease of another, we have a -ve association of relationship.

Types of STATISTICAL RELATIONSHIP

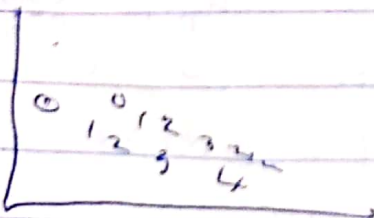
=> Scatter diagram



=> Positive diagram



=> Negative diagram



Karl Pearson's Product

Moment Correction Coefficient

It is the most popular mathematical method of quantifying or measuring correlation. It is always denoted by r and is given by

$$r = \frac{\text{Covariance}(x, y)}{\sigma_x \cdot \sigma_y} = \frac{\sum xy}{\sum x \sum y}$$

$$= \frac{\sum xy}{N(\sigma_x \sigma_y)}$$

$$r = \frac{\text{Covariance}(x, y)}{\sigma_x \sigma_y} = \frac{\sum xy}{\sum x \sum y}$$

$$= \frac{\sum xy}{N(\sigma_x \sigma_y)}$$

$$N = \text{Population size}$$

$$\sigma_x =$$

where σ_x, σ_y are standard deviation of x and y respectively.

N = Population size.

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \cdot \sum y^2}}$$

$$r = \frac{N \sum xy - \sum x \sum y}{\sqrt{(N \sum x^2 - (\sum x)^2) (N \sum y^2 - (\sum y)^2)}}$$

$$\sqrt{\underbrace{(N \sum x^2 - (\sum x)^2)}_{\sigma_x^2} \underbrace{(N \sum y^2 - (\sum y)^2)}_{\sigma_y^2}}$$

r is b/w 0 and 1

When its close to 1, its relationship is HIGH but when it close to 0, its relationship is LOW.

Reading

CHI SQUARE

→ A die is thrown 270 times and results of these throws are given below.

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not.
Sol.

Null hypothesis H_0 - Die is unbiased.

Under H_0 , expected frequencies for each digit = $\frac{\Sigma F}{n}$

$$i.e. \frac{40+32+29+59+57+59}{6} = \frac{276}{6}$$

$$E = \frac{276}{6} = 46$$

∴ the expected Value for each of them is 46 i.e.

O_i	40	32	29	59	57	59
E_i	46	46	46	46	46	46
$O_i - E_i$	6	-14	-17	13	11	13
$(O_i - E_i)^2$	36	196	289	169	121	169

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{980}{46} = 21.30$$

degree of freedom = $n-1 = 6-1 = 5$

from table,

At 0.05 i.e. 5% with $n=5$, $\chi^2 = 11.07$

Since Value from table $<$ cal. χ^2

i.e. $11.07 < 21.30$ ∴ H_0 is rejected

i.e. die is not unbiased i.e. die is biased

Note, if Value from table $>$ Cal. χ^2

then H_0 is accepted i.e. die is unbiased

→ Records taken of a number of male and female births is 800 families having four children are as follows:

No. of males birth	0	1	2	3	4
No. of female birth	4	3	2	1	0
No. of families	32	178	290	236	94

Test whether the data are consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of female birth, namely, $p=q=\frac{1}{2}$.

Sol

H_0 : The data are consistent with hypothesis of equal probability for male and female birth, i.e. $p=q=\frac{1}{2}$

Using binomial distribution to calculate theoretical frequency.

$$N(r) = N \times P(X=r)$$

Where N is total frequency.

$N(r)$ is no. of families with r male children

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

where p and q are probability of male and female birth, respectively, n is no. of children.

$N(0)$ = No. of families with 0 male children

$$i.e. N(0) = 800 \times P(X=0)$$

$$i.e. N(0) = 800 \times {}^4C_0 \left(\frac{1}{2}\right)^4$$

$$= 800 \times 1 \times \frac{1}{2^4} = 50$$

$$N(1) = 800 \times {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 = 200$$

$$N(2) = 800 \times {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 300$$

$$N(3) = 800 \times {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = 200$$

$$N(4) = 800 \times {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = 50$$

Observed freq. O_i	32	178	290	236	94
Expected freq. E_i	50	200	300	200	50
$(O_i - E_i)^2$	324	484	100	1296	1936
$\left(\frac{O_i - E_i}{E_i}\right)^2$	6.48	2.42	0.333	6.48	38.72

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 54.433$$

from degree of freedom $df = 5 - 1 = 4$

from table,

~~0.05 or 5% again~~ 4 against 0.05 + 25%
On Chi square table, ~~2~~

It is 9.488 $\approx$ 9.49

Since $9.49 < 94.433$, H_0 is rejected.

i.e. data are not consistent with the

hypothesis that a binomial law holds and that the chance of a male birth is not equal to that of a female birth.

$\Rightarrow$ Verify whether Poisson distribution can be assumed from the data given below;

No. of defect	0	1	2	3	4	5
Frequency	6	13	13	8	4	3

Sol, using Poisson.

H_0 : Poisson fit is a good fit to the data.

Mean of the given distribution = $\frac{\sum f_i x_i}{\sum f_i} = \frac{94}{47} = 2$

To fit a Poisson distribution we require

m. parameter $m = \bar{x} = 2$.

By Poisson distribution, the frequency of success is

$N(x) = N \times e^{-m} \cdot \frac{m^x}{x!}$, N is the total frequency

$$N(0) = 47 \times e^{-2} \cdot \frac{(2)^0}{0!} = 6.36 \approx 6$$

$$N(1) = 47 \times e^{-2} \cdot \frac{(2)^1}{1!} = 12.72 \approx 13$$

$$N(2) = 47 \times e^{-2} \cdot \frac{(2)^2}{2!} = 12.72 \approx 13$$

$$N(3) = 47 \times e^{-2} \cdot \frac{(2)^3}{3!} = 3.48 \approx 4$$

$$N(4) = 47 \times e^{-2} \cdot \frac{(2)^4}{4!} = 1.74 \approx 2$$

$$N(5) = 47 \times e^{-2} \cdot \frac{(2)^5}{5!} = 0.696 \approx 1$$

x	0	1	2	3	4	5
f_i	6	13	13	8	4	3
e_i	6.36	12.72	12.72	8.48	4.24	1.696
$\frac{(O_i - E_i)^2}{E_i}$	0.2037	0.00616	0.00616	0.0216	0.0135	1.0026

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 1.2864$$

Conclusion

The Cal. value of $\chi^2 = 1.2864$.

Tabulated value of χ^2 at 5% level of significance for $\gamma = 6 - 2 = 4$ df is 9.49. Since the Cal. value of χ^2 is less than that of tabulated value.

H_0 is accepted i.e. Poisson distribution provides a good fit to data.

The theory predicts the proportion of beans in four groups G_1, G_2, G_3, G_4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans and four groups were 882, 313, 287 and 118. Does the experimental result support the theory?

Sol
 H_0 : The experimental result supports the theory i.e. there is no significant diff b/w observed and theoretical freq. Under H_0 the theoretical frequency can be Cal. as follows;

$$E(G_1) = \frac{1600 \times 9}{16} = 900$$

$$E(G_2) = \frac{1600 \times 3}{16} = 300$$

$$E(G_3) = \frac{1600 \times 3}{16} = 300$$

$$E(G_4) = \frac{1600 \times 1}{16} = 100$$

To Cal. the value of χ^2 .

Observed freq. O_i	882	313	287	118
Expected freq. E_i	900	300	300	100
$\frac{(O_i - E_i)^2}{E_i}$	0.36	0.5633	0.5633	1.62

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 4.7266$$

Conclusion: H_0

Table value of χ^2 at 5% level of significance of 3 df is 7.815. Since the Cal. value of χ^2 is less than that of tabulated value. Hence H_0 is accepted i.e. the experimental result supports the theory.

2-6-2021

class

LINEAR REGRESSION

used for prediction of

This ~~looks for~~ dependent variable x and tends used for ~~prediction~~ dependent variable y

$$y = a + bx \quad \text{--- (i)}$$

$$\sum y = an + b \sum x \quad \text{--- (ii)}$$

$$\sum xy = a \sum x + b \sum x^2 \quad \text{--- (iii)}$$

Where a & b = Constants.

n = no. of sample.

$$y = a + bx_1 + cx_2 \quad \text{--- (1)}$$

$$y = ax + b \sum x_1 + c \sum x_2 \quad \text{--- (2)}$$

$$\sum xy = a \sum x + b \sum x_1^2 + c \sum x_1 x_2 \quad \text{--- (3)}$$

$$\sum x_2 y = a \sum x_2 + b \sum x_1 x_2 + c \sum x_2^2 \quad \text{--- (4)}$$

Example

x	y	xy	x^2
4	5	20	16
6	4	24	36
7	5	35	49
8	6	48	64

$$n = 4, \sum xy = 25, \sum x = 20, \sum xy = 127, \sum x^2 = 165$$

Then Sub to

$$y = a + bx$$

$$\sum y = an + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$\sum y = an + b \sum x$$

$$20 = 4a + 25b \quad \text{--- (1)}$$

$$\sum xy = a \sum x + b \sum x^2$$

$$\text{ie } 127 = 23a + 165b \quad \text{--- (ii)}$$

$y = a + bx$ --- (1)
then sub a and b , data d eqn.

ANALYSIS OF VARIANCE ONE WAY TEST

Let Consider sample data as presented below.

female = 4, 3, 4, 3, 5, 7, 6, 9, 2, 5, 3.

male = 2, 4, 4, 3, 4, 8, 7, 8, 9, 9, 2.

$$t\text{-test } (t) = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

μ = mean of population. $\frac{s}{\sqrt{n}}$

s = standard deviation of sample.
 n = no. of sample.

To find mean $\bar{x}$

$$df = n - 1 = 11 - 1 = 10.$$

note μ = must be given.

If the male and female is rejected
u sum d two ie F test

$$F_{\alpha} = \frac{S^2_1}{S^2}$$

STATISTICS (Bingr. Dr. Mrs. Polashade)
Tayase.

STATISTICAL INFERENCE

2 Types of STATISTICS

$\Rightarrow$ Descriptive statistics.

$\Rightarrow$ Inferential statistics.

example of population

A d males produce in a batch in a day

3 main underline ideas of a sample

$\Rightarrow$ A sample is likely to be a good representation of a population.

$\Rightarrow$

Types of Sampling

⇒ Random Sampling ⇒ no bias, no preference. It is used to eliminate bias.

⇒ Stratified Sampling ⇒ The population is divided into two called strata.
i.e. dividing a population into groups and pick some same no of samples from each group.

⇒ Cluster sampling ⇒ When samples are taken from an already existed groups.

⇒ Systematic Sampling ⇒ Dividing a group according to the range.

Values of Taken Sample

⇒ A sample is a good representation of a population

⇒

9-6-2021

SAMPLING DISTRIBUTION

Parameter is a population data. It is a quantity that measure / describe a population.

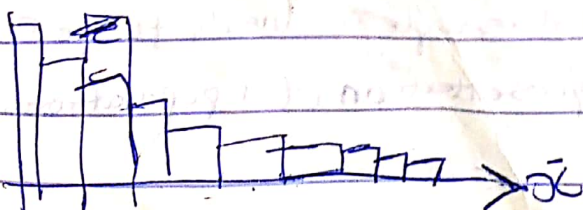
e.g. population mean,

Parameter	Statistics
⇒ population mean μ	Sample mean $\bar{x}$
⇒ population variance σ^2	" Variance
⇒ " standard deviation	" standard deviation
⇒ " proportion	" proportion

If u find d means of each samples.
and take 3 out of 10 population,

$$\text{i.e. } \binom{10}{3} = {}^{10}C_3 = 120$$

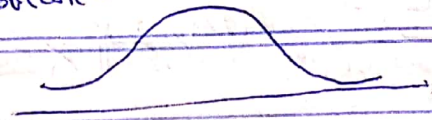
If u draw their histogram



This is 10 histograms

the means

at the long run, if u do it to 120 histogram, it give a normal distribution.



base of disabilities in sample mean, we have standard error.

$$\text{Mean } \bar{x} = E\left(\frac{x}{n}\right) \rightarrow \text{mean} = E\frac{x}{n}$$

$$\text{Variance} = E\left(\frac{x - \bar{x}}{n}\right)^2 \rightarrow \frac{\sigma^2}{n} \rightarrow \text{standard error}$$

$$\text{standard deviation} = \sqrt{\frac{E(x - \bar{x})^2}{n}} \rightarrow \sqrt{\frac{\sigma^2}{n}}$$

$$\text{Population} = \sqrt{\frac{n \lambda (1 - \lambda)}{n}}$$

Properties of Sampling Distribution

For any no of samples n d cent of mean distribute is also coincident with the mean of population and spread of mean distribution decreases as n increases.

Larger samples lower distribution

Let $\bar{x}$ = means of d observation in a random sample of size n from a population having mean μ and standard deviation σ . The mean value of the $\bar{x}$ distribution of μ and d standard deviation of $\bar{x}$ distribution by $\sigma_{\bar{x}}$. The following rules hold.

Sample dis. for Sample mean

Rule 1:

$$\mu_{\bar{x}} = \mu$$

Rule 2: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$\sigma_{\bar{x}}$ = Standard deviation of the sample

Rule 3: When population distribution is normal, the sampling distribution of $\bar{x}$ is also normal for any sample size.

Rule 4: Central limit theory. When n is sufficiently large, the sampling distribution is well approximated by a normal curve even when the population distribution is not itself normal.

Sample distribution for sample proportion
 either a random ~~exam~~

π = proportion,

parameter

Statistic.

π = population

p-sample proportion

p = no of successes in the sample

no of sample (n)

① properties of sample proportion

⇒ The sample distribution of p is centred at π i.e. $\mu_p = \pi$.

p is an unbiased statistic for estimating π

⇒ The standard deviation of p

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

③ As long as n is large, and $(n\pi \geq 10)$ and $n(1-\pi) \geq 10$

[Reading]

CORRELATION AND REGRESSION

RELATIONSHIP
 a relationship btw two series. e.g height and weight of a group of people. weight increases with increase in height. Such relationship is called STATISTICAL RELATIONSHIP.

TYPES OF DISTRIBUTION

$$x = \frac{\sum x}{n}$$

⇒ UNIVARIATE DISTRIBUTION ⇒ This is a distribution in which there exist only one variable. e.g height of students in a class.
 ⇒ BIVARIATE DISTRIBUTION ⇒ This involves two variables e.g height and weight of a class.

COVARIANCE

If $(x_1, y_1), (x_2, y_2), (x_3, y_3) \dots (x_n, y_n)$

The covariance btw x and y is;

$$\text{COV}(x, y) = \frac{(x_1 - \bar{x})(y_1 - \bar{y}) + (x_2 - \bar{x})(y_2 - \bar{y})}{n}$$

$$\text{i.e. } \text{COV}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

EXAMPLE

⇒ Cal the Covariance of the following pairs of observations of two variates.

$(1, 4), (2, 2), (3, 4), (4, 8), (5, 9), (6, 12)$
 $n = 6$

$$\sum x_i = 1 + 2 + 3 + 4 + 5 + 6 = 21$$

$$\sum y_i = 4 + 2 + 4 + 8 + 9 + 12 = 39$$

$$\begin{aligned} \sum x_i y_i &= (1 \times 4) + (2 \times 2) + (3 \times 4) + (4 \times 8) + (5 \times 9) + (6 \times 12) \\ &= 4 + 4 + 12 + 32 + 45 + 72 \\ &= 169 \end{aligned}$$

$n = 6$ i.e. 6 brackets.

$$\text{COV } x, y = \frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n} \cdot \frac{\sum y_i}{n}$$

$$\begin{aligned} \therefore \text{COV } x, y &= \frac{169}{6} - \frac{21}{6} \times \frac{39}{6} \\ &= \frac{169}{6} - \frac{91}{4} = \frac{65}{12} \end{aligned}$$

$$\therefore \text{COV } x, y = \frac{65}{12}$$

CORRELATION

CORRELATION is a situation whereby when variable x and y are so related that increase in one is as a result of increase or decrease in the other. e.g.

The yield of crop varies with the amount of rainfall. i.e. variation (increase or decrease) in rainfall determines the quantity of crop that will yield.

TYPES

1 Positive Correlation $\Rightarrow$ A Correlation is

said to be +ve if the increase in one variable (x) results in a corresponding increase in value of other variable (y) on an average.

OR

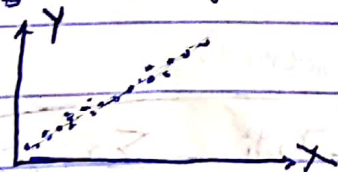
If the decrease in value of one variable x results in corresponding decrease in value of other variable y on an average.

2 Negative Correlation $\Rightarrow$ This is when the increase in value of one variable x results in a corresponding decrease in the values of other variable y .

OR

If the decrease in value of one variable x results in a corresponding increase in the values of other variable y .

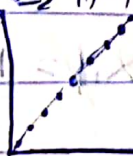
3 LINEAR CORRELATION $\Rightarrow$ When all plotted points lie approximately on a straight line.



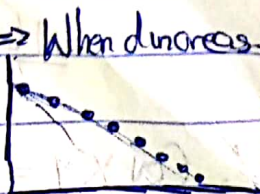
4 PERFECT CORRELATION $\Rightarrow$

A Positive perfect correlation $\Rightarrow$ If the increase in one variable x is proportional to increase in other variable y .

The graph will be straight line.



B Negative perfect correlation $\Rightarrow$ When a decrease in one variable is proportional to increase in the other variable. The graph is also a straight line.



KARL PEARSON'S COEFFICIENT OF CORRELATION

$$r = \frac{\sum XY}{\sqrt{(\sum X^2)(\sum Y^2)}} = \frac{P}{\sigma_x \sigma_y}$$

$$= \frac{\text{Covariance}(x, y)}{\sqrt{\text{Variance}(x)} \sqrt{\text{Variance}(y)}}$$

where $X = x - \bar{x}$, $Y = y - \bar{y}$

$$\text{Covariance}(P) = \frac{\sum XY}{n}$$

and σ_x, σ_y , being standard deviation of these series

EXAMPLES

$\Rightarrow$ Cal. the coefficient of correlation between x and y series from the following data:

$$\sum (x - \bar{x})^2 = 136 \quad \sum (y - \bar{y})^2 = 138$$

$$\sum (x - \bar{x})(y - \bar{y}) = 122$$

Sol

$$\text{Since } P = \frac{\sum (x - \bar{x})(y - \bar{y})}{n} = \frac{\sum XY}{n}$$

Also note; $(x - \bar{x})$ = mean deviation

$$(x - \bar{x})^2 = \text{Variance}$$

$$\sqrt{V} = \sqrt{(x - \bar{x})^2} = \text{Standard deviation}$$

Formula to use;

$$r = \frac{\sum XY}{\sqrt{\sum X^2} \sqrt{\sum Y^2}} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}}$$

$$\text{i.e. } r = \frac{\sum (x - \bar{x})(y - \bar{y})}{(\text{Standard deviation of } x)(\text{Standard deviation of } y)}$$

$$r = \frac{122}{(\sqrt{136})(\sqrt{138})}$$

$$r = \frac{122}{11.16 \times 11.75} = \frac{122}{130.32} = 0.89$$

ANALYSIS
(Coefficient of correlation of grouped data)

x	y	$\bar{x} = 65$ $X = x - \bar{x}$	$\bar{y} = 66$ $Y = y - \bar{y}$	$X^2 = (x - \bar{x})^2$	$Y^2 = (y - \bar{y})^2$	$XY = (x - \bar{x})(y - \bar{y})$
78	84	13	18	169	324	234
36	51	-29	-15	841	225	-435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	-240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	-24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	-494
$\Sigma x = 650$	$\Sigma y = 660$	$\Sigma X = 0$	$\Sigma Y = 0$	$\Sigma X^2 = 5398$	$\Sigma Y^2 = 2224$	$\Sigma XY = 2704$

$$r = \frac{122}{11.66 \times 11.75} = \frac{122}{137.005} = 0.89$$

$$\therefore r = 0.89$$

$\Rightarrow$ Ten students got the following % of marks in Economics and Statistics.

Roll no	1	2	3	4	5	6	7	8	9	10
Marks in Economics	78	36	98	25	75	82	90	62	65	39
Marks in Statistics	84	51	91	60	68	62	86	58	53	47

Cal. the coefficient of correlation.

Sol.

Let x = marks in Economics

y = marks in Statistics

$$\bar{x} = \frac{78+36+98+25+75+82+90+62+65+39}{10}$$

$$\bar{x} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{84+51+91+60+68+62+86+58+53+47}{10}$$

$$\bar{y} = \frac{660}{10} = 66$$

$$\therefore \sum x^2 = 5398, \quad \sum y^2 = 2224$$

$$\sum xy = 2704$$

$$r = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}} = \frac{2704}{\sqrt{5398} \sqrt{2224}}$$

$$r = \frac{2704}{73.4 \times 47.1} = \frac{2704}{3457} = 0.78$$

$$\therefore r = 0.78$$

REGRESSION

Line of Regression

A line of regression is the straight line which gives the best fit in the least square sense to the given frequency. The line is a straight line.

EQUATIONS TO THE LINES OF REGRESSION

NOTE.

Regression of y on x is;

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

Regression of x on y is;

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

The slope of the line of regression;

$$b = r \frac{\sigma_y}{\sigma_x}$$

$$\therefore b_{yx} = r \frac{\sigma_y}{\sigma_x}, \quad b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

i.e. b_{yx} and b_{xy} are called coefficient of regression.

$$b_{yx} \cdot b_{xy} = \left(r \frac{\sigma_y}{\sigma_x} \right) \left(r \frac{\sigma_x}{\sigma_y} \right) = r^2$$

EXAMPLES

$\Rightarrow$ Find the correlation coefficient between x & y , when the lines of regression are;
 $20x - 9y + 6 = 0$, $x - 2y + 1 = 0$

Sol.

(Let line of regression of x on y be $20x - 9y + 6 = 0$
 " " " " " y on x " $x - 2y + 1 = 0$)

$$20x - 9y + 6 = 0 \Rightarrow 20x = 9y - 6 \Rightarrow x = \frac{9}{20}y - \frac{3}{10}$$

$$\text{i.e. } b_{xy} = \frac{9}{20}$$

$$x - 2y + 1 = 0 \Rightarrow y = \frac{1}{2}x + \frac{1}{2}$$

$$\therefore b_{yx} = \frac{1}{2}$$

Since $r^2 = b_{xy} \cdot b_{yx}$.

$$r = \sqrt{\frac{9}{2} \times \frac{1}{2}} = \sqrt{\frac{9}{4}} = \frac{3}{2} > 1$$

Since $\frac{3}{2} > 1$, this is incorrect because the answer must be lesser than 1.
~~Switch~~ Changing $\therefore$ The chosen regression line is incorrect.

~~Switch~~ Interchanging the two eqns;

Let the line of regression of y on x be $x - 9y + 6 = 0$.

$$y = \frac{2}{9}x + \frac{6}{9}$$

i.e. $b_{yx} = \frac{2}{9}$

Let the line of regression of x on y be $x - 2y + 1 = 0$

$$x = 2y - 1$$

i.e. $b_{xy} = 2$.

$$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{2 \times \frac{2}{9}} = \frac{2}{3}$$

$\therefore$ The correlation coefficient between x and y is $\frac{2}{3}$.

$\Rightarrow$ The regression eqns calculated from a given set of observations for two random variables are;

$$x = -0.4y + 6.4, \quad y = -0.6x + 4.6$$

Cal (a) Mean of x's ($\bar{x}$)

(b) mean of y's ($\bar{y}$)

(c) The correlation coefficient between x and y (r)

Sol.

$$x = -0.4y + 6.4 \quad \text{--- (1)}$$

$$y = -0.6x + 4.6 \quad \text{--- (2)}$$

Since eqn (1) and (2) passes through the

point $(\bar{x}, \bar{y})$ (2) and (6)

$\therefore$ eqn becomes.

$$\bar{x} = -0.4\bar{y} + 6.4 \quad \text{--- (3)}$$

$$\bar{y} = -0.6\bar{x} + 4.6 \quad \text{--- (4)}$$

Solving eqn 3 and 4 simultaneously

$$\bar{x} = 6, \quad \bar{y} = 1$$

(c)

from eqn (1) The coefficient of x on y $= b_{yx} = r \frac{\sigma_y}{\sigma_x} = -0.4$ --- (5)

from eqn (2)

The coefficient of y on x $= b_{xy} = r \frac{\sigma_x}{\sigma_y} = -0.6$

The correlation coefficient } r,

$$r^2 = b_{xy} \cdot b_{yx}$$

$$r^2 = r \frac{\sigma_x}{\sigma_y} \cdot r \frac{\sigma_y}{\sigma_x}$$

$$r^2 = (-0.4)(-0.6) = 0.24$$

$$r = \sqrt{0.24} = \pm 0.49$$

$$r = \pm 0.49$$

from eqn (5) and (6), σ_x and σ_y are (always) +ve, so r is negative.

$$\text{i.e. } r = -0.49$$

16-6-2021

CLASS

CONFIDENCE INTERVAL

$\Rightarrow$ Introduction to Confidence interval for one mean (σ known) (youtube)

Channel $\Rightarrow$ jb statistics.

$\Rightarrow$ Confidence interval and margin error Ap Statistics.

Channel; Khan Academy.

$\Rightarrow$ Solved problems and Submit.

To be submitted on wed 23, 2021

Heading

SAMPLING OF VARIABLES

POPULATION $\Rightarrow$ This is a group of individuals under study. It is also known as **UNIVERSE**.

SAMPLING

A part selected from the population is called **SAMPLE**. The process of selection of a sample is **SAMPLING**.

RANDOM SAMPLING $\Rightarrow$ This is one in which each member of population has an equal chance of being included in it.

N means there are diff samples of size n that can be picked up from a population of size N .

PARAMETERS AND STATISTICS

The statistical constants of the population such as mean (μ), standard deviation (σ) are called **Parameters**. Parameters are denoted by **Green words**.

The mean ($\bar{x}$), standard deviation (s) of a sample are known as **STATISTICS**. Statistics are denoted by **Brown letters**.

Symbols for population and samples

Population
Samples

Population	Sample
Parameters	Statistics
Population size = N	Sample size = n
Population mean = μ	Sample mean = $\bar{x}$
Population standard deviation = σ	Sample standard deviation = s
Population proportion = p	Sample proportion = $\bar{p}$

AIMS OF SAMPLES

STATISTICAL INFERENCE $\Rightarrow$ The logic of the sample theory is the **Logic of Induction**. In induction, we pass from a particular (sample) to general (population). This type of generalization is here is known as **STATISTICAL INFERENCE**.

TYPES OF SAMPLING

$\Rightarrow$ Purposive sampling

$\Rightarrow$ Random sampling

$\Rightarrow$ Stratified sampling

$\Rightarrow$ Systematic sampling.

SAMPLING DISTRIBUTION

From a population N , a sample are drawn of equal size n . Find out the **mean** of each sample. The means of samples are not equal. The means with their respective frequencies are grouped. The frequency distribution so formed is known as **SAMPLING DISTRIBUTION of the mean**. Similarly, sampling distribution of standard deviation we can have.

STANDARD ERROR (S.E)

This is the standard deviation of the sampling distribution. For assessing a diff b/w expected value and **observed value**, standard error is used. Reciprocal of standard error is **PRECISION**.

SAMPLING DISTRIBUTION OF MEANS FROM INFINITE POPULATION.

Let x = random variable denoting the measurement of the characteristics.

Let μ = population mean.

Variance of x , $\text{Var}(x) = \sigma^2$

Expected value of x , $E(x) = \mu$

Then the Sample mean ($\bar{x}$) is a sum of n random Variables $x_1, x_2, \dots, x_n$ each being divided by n .

i.e. $x_1, x_2, \dots, x_n$ are independent random Variable from infinitely large population.
Now:

Expected of x_1 , $E(x_1) = \mu$, Variance of x_1 , $Var(x_1) = \sigma^2$

Expected of x_2 , $E(x_2) = \mu$, Variance of x_2 , $Var(x_2) = \sigma^2$

$\therefore$ Sample Expected mean, $E(\bar{x}) = E\left[\frac{x_1 + x_2 + \dots + x_n}{n}\right]$

$$E(\bar{x}) = \frac{1}{n}E(x_1) + \frac{1}{n}E(x_2) + \dots + \frac{1}{n}E(x_n)$$

Since $E(x_n) = \mu$

$$\therefore E(\bar{x}) = \frac{1}{n}\mu + \frac{1}{n}\mu + \dots + \frac{1}{n}\mu$$

$$= \mu$$

i.e. $E(\bar{x}) = \mu$

$\therefore$ The expected value of sample mean $E(\bar{x})$ is same as population mean μ .
For Variance:

$$\text{Variance of Sample mean, } Var(\bar{x}) = Var\left[\frac{x_1 + x_2 + \dots + x_n}{n}\right]$$

$$Var(\bar{x}) = Var\left(\frac{x_1}{n}\right) + Var\left(\frac{x_2}{n}\right) + \dots + Var\left(\frac{x_n}{n}\right)$$

$$Var(\bar{x}) = \frac{1}{n}Var(x_1) + \frac{1}{n}Var(x_2) + \dots + \frac{1}{n}Var(x_n)$$

Since $Var(x_n) = \sigma^2$,

$$\therefore Var(\bar{x}) = \frac{1}{n}\sigma^2 + \frac{1}{n}\sigma^2 + \dots + \frac{1}{n}\sigma^2$$

$$\therefore Var(\bar{x}) = \frac{\sigma^2}{n}$$

$\therefore$ The Variance of Sample mean ($Var(\bar{x})$) is the Variance of population mean (σ^2) divided by the Sample size (n)

STANDARD ERROR OF THE MEAN

This is also known as STANDARD DEVIATION ($\frac{\sigma}{\sqrt{n}}$) i.e. $\sqrt{\text{Variance}}$

$$\therefore \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

SAMPLING WITH REPLACEMENT

When sampling is done with replacement, so that the population is back to same form by the next sample member is picked up, we have;

$$E(\bar{x}) = \mu$$

$$Var(\bar{x}) = \frac{\sigma^2}{n}$$

$$\text{Standard deviation, } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

SAMPLING WITHOUT REPLACEMENT FROM FINITE POPULATION.

$$E(\bar{x}) = \mu$$

$$Var(\bar{x}) = \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

$$\text{Also } \sigma_{\bar{x}} = \sqrt{Var(\bar{x})}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

If $\frac{n}{N}$ is very small,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \text{ approximately.}$$

SAMPLING FROM NORMAL POPULATION

If $x \sim N(\mu, \sigma^2)$ then it follows that

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Ass

Introduction to Confidence interval for one mean (σ known) youtube channel $\Rightarrow$ JB Statistics.

SOL

Note $\bar{x}$ is a point estimator of μ
 $\nwarrow$ sample mean $\nearrow$ population mean

The confidence interval;

$$\bar{x} \pm \text{margin of error}$$

for confidence interval method to be reasonable we require;

A simple random sample from the population of interest (It is very important regardless of sample size).

of normal distribution population

(This is not important for large sample size).

Also, population standard deviation σ must be known.

Confidence Interval formula;

Confidence interval for $\mu = A(1-\alpha)100\%$

$$\bar{x} \pm \underbrace{\frac{z_{\alpha/2}}{2} \times \frac{\sigma}{\sqrt{n}}}_{\text{margin of error of confidence interval}}$$

$$\sigma_{\bar{x}} = \frac{\text{Standard deviation of sampling distribution}}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$(1-\alpha)100\%$ is a confidence interval level which is commonly 95%.

Note α is gotten from the confidence interval level.

e.g. if confidence interval level = 95%

i.e. 95% = 0.95.

$$\alpha = \frac{100\% - 95\%}{2} = \frac{5\%}{2} = \underline{2.5\%}$$

$$\therefore \alpha = 0.025$$

To Z-table, $z(\frac{\alpha}{2}) = 1.96$

i.e. $0.95 + 0.025 = 0.975$.

on Z-table, 0.9750 is under 1.9 under 0.06 i.e. $1.9 + 0.06 = 1.96$.

Example

The ratio of the length of the second digit (index finger) to that of the fourth digit (ring finger), known as the 2D:4D ratio is often studied by researchers.

Sol

A study investigated the 2D:4D ratio in women of European descent at a large college.

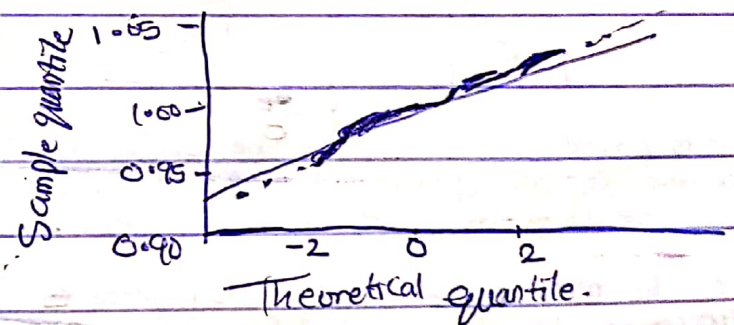
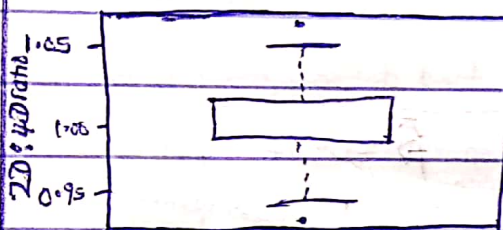
A sample of 135 women yielded an average 2D:4D ratio of 0.988.

i.e. $(\bar{x}) = 0.988$.

What is the 95% confidence interval for the population mean?

Suppose it is known that $\sigma = 0.025$.

The box plot and gentile quantile plot



$$\bar{x} = 0.988 \quad \sigma = 0.025, \quad n = 135$$

What is a 95% confidence interval for the population mean?

$$\bar{x} \pm \frac{z_{\alpha/2}}{2} \times \frac{\sigma}{\sqrt{n}}$$

Note $z(\frac{\alpha}{2}) =$

Substituted value

Note $z(\frac{\alpha}{2}) = 1.96$

$$0.988 \pm 1.96 \times \frac{0.025}{\sqrt{135}}$$

$$= 0.988 \pm 0.0047$$

$$= (0.988 + 0.0047, 0.988 - 0.0047)$$

$$= (0.993, 0.983)$$

- We can be 95% Confident that the population mean lies in this interval (0.983, 0.993)
- We can be 95% Confident that the true mean is 0.983 and 0.993.

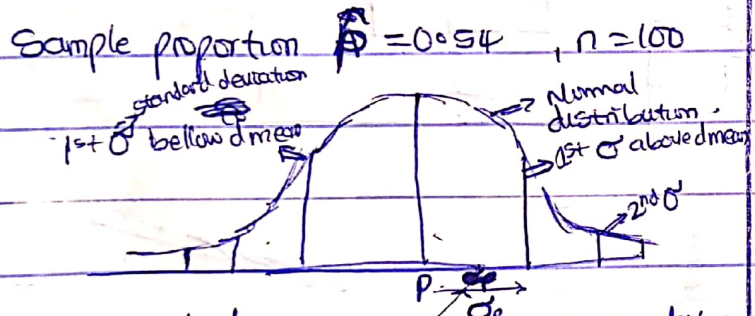
Q2

Confidence intervals and margin of error AP statistics Khan Academy.
Channel => Khan academy.

QOL

In an election betw A and B Candidate, a Population Size of $N=100$, People who support Candidate A is 54% and people who support " B is 46%. Sampling distribution of sample proportions (for $n=100$).

To find d Sampling distribution.



It is normal distribution bco we assume that the true proportion (Proportion that support A) isn't too close to zero or not too close to one.

i.e. The mean of this sampling distribution is going to be d actual population proportion (p)

i.e. $\bar{x} = p$
 standard deviation for sample proportion for this sampling distribution

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

For $n=100$, $\hat{p} = 0.54$

i.e. To find the position where 0.54 lies on the Normal distribution above.

Note Actual population p is unknown.

Q5) What is d probability that $\hat{p} = 0.54$ is within $2\sigma_{\hat{p}}$ of p?

It will be approximately 95%

i.e. there is a 95% prob. that population proportion p is within $2\sigma_{\hat{p}}$ of $\hat{p} = 0.54$.

Since $p = ? = \text{true proportion}$

Using anoda formula for estimating for population proportion.

Since Sample proportion $\hat{p} = 0.54$

Using Standard Error (SE) formula

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$SE_{\hat{p}} = \sqrt{\frac{0.54(1-0.54)}{100}} = 0.05$$

∴ with 95% Confidence, betw $[0.54 - (2 \times 0.05) = 0.44]$ and $[0.54 + (2 \times 0.05) = 0.64]$

0.44 and 0.64 votes support A.

The Confidence Interval = 0.44 to 0.64.

The related idea to the confidence interval is d notion of margin of error.

$$\text{Margin of error} = 2 SE_{\hat{p}}$$

$$\text{i.e. Margin of Error} = 2 \times 0.05 = 0.1$$

$$\text{Margin of Error below sample proportion is } 0.44 - 0.1 = 0.34$$

$$\text{Margin of Error above sample proportion is } 0.64 + 0.1 = 0.74$$

To tighten up the intervals on average

To tighten up the intervals on average?

Lower the margin of error by increasing the no of size of population (sample size).

23-6-2021

CLASS

NOTE. Sampled distribution problem

The average male drinks 2L of water when active - outdoors with (with a standard deviation of 0.7L) you are planning a full day nature trip for 50 men and will bring 110L of water. What is the probability that you will run out.

SOL

Population mean $\mu = 2L$

$\sigma = 0.7L$ $n = 50$

$p > 110L$

Sample mean $\bar{x} = \frac{110}{50} = 2.2L$

$$\sigma_{\bar{x}}^2 (\text{Variance}) = \frac{\sigma^2}{n} = \frac{(0.7)^2}{50}$$

$$\sigma_{\bar{x}} (\text{S.D} = \sqrt{\text{Variance}} = \frac{\sigma}{\sqrt{n}} = \frac{0.7}{\sqrt{50}}$$

$$\sigma_{\bar{x}} = 0.099$$

note if $n > 30$, it means

The diagram is tending to normal distribution
to find the no of standard deviation at which it is far from the centre

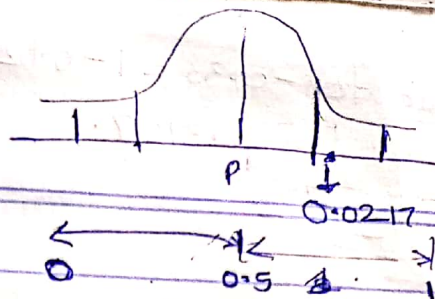
$$\frac{\bar{x} - \mu}{\sigma} = \frac{2.2 - 2}{0.099} = 2.02$$

i.e. after two standard deviation.

from Z-table, $2.02 = 0.9783$

Since the whole diagram is 1.

To find the prob. is $1 - 0.9783$
 $= 0.0217$



Prob that $p > 110L = 2\%$

Suppose samples of 100 items were drawn from a population with a mean of 200 and a standard deviation 30.

What proportion of sample will have mean,

(a) greater than 202

(b) less than 199

(c) between 198.5 and 203

SOL

$n = 100$, $\mu = 200$, $\sigma = 30$

$$(i) Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

Since $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$$Z = \frac{202 - 200}{\frac{30}{\sqrt{100}}} = \frac{2}{3} = 0.67$$

under Z table, $0.67 = 0.7486$
i.e. 0.6 under 0.07

$$\text{posn} = 1 - 0.7486 = 0.2514$$

Prob. = 25%

(ii)

$$Z = \frac{199 - 200}{\frac{30}{\sqrt{100}}} = \frac{-1}{3} = -0.33$$

Since answer is negative, ~~0.33~~

under -ve table, $-0.33 = 0.6293$

but if the table was given,

check 0.3 under 0.03 in the table
 $= 0.6293$

Actual z value for $-0.33 = 1 - 0.6293$
 $= 0.3707$.

Prob - = 37%

(ii)

$$Z = \frac{195.5 - 200}{3} = \frac{-1.5}{3} = -0.50$$

under negative table, 0.5 under

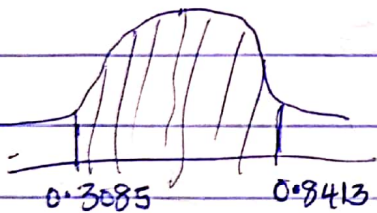
$$Z_{-0.5} = 0.3085$$

Also for 203.

$$Z = \frac{203 - 200}{3} = 1.00$$

under +ve table, 1.0 under 0.00

$$Z_{1.0} = \underline{0.8413}$$



Since $\begin{matrix} * \\ 0 \end{matrix}$ $\begin{matrix} * \\ 0.5 \end{matrix}$ $\begin{matrix} * \\ 1 \end{matrix}$
 To get actual point,

$$0.8413 - 0.3085 = 0.5328$$

$$= \underline{53\%}$$

NOTE There are two types of z tables.

1) The one of 1 like above to identify first row and first column under zero will be 0.5

2) The one with 0.5 to identify, first row first column, u will see 0.0000

Confidence interval

→ The ratio of the length of the second digit (Index finger) to that of the fourth digit (Ring finger) known as the 2D:4D is often studied by researchers. A study investigated the 2D:4D at a large College.

A sample of 135 women yielded an average of 2D:4D ratio 0.988. What is d Confidence Interval for the population mean? Suppose it is known that $\sigma = 0.028$.

sol.

$$\bar{x} = 0.988, \sigma = 0.028, n = 135$$

$$\text{Since } (1 - \alpha) = 95\%$$

$$\text{i.e. } \alpha = 5\% = 0.05$$

Recall that ~~$\bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$~~

$$\bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

$$= 0.988 \pm \frac{Z(0.028)}{2} \times \frac{0.028}{\sqrt{135}}$$

Assumptions in Confidence intervals are 95%, 90% and 99.9%

Ass

A sample poll of 100 voters chosen at random from all voters in a given district indicated that 55% of them were in favour of a particular candidate, find (a) 95% (b) 99% (c) 99.73% Confidence limits for the proportion of all the voters in favour of this candidate.

30-8-2021

CLASS

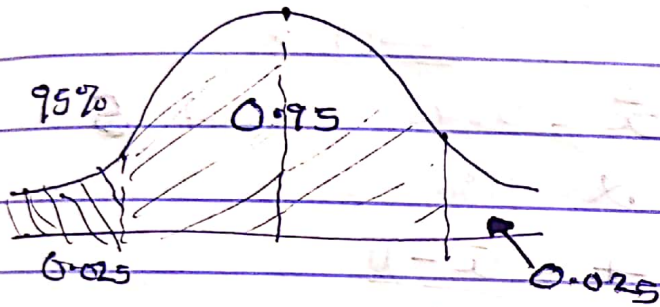
CONFIDENCE INTERVAL

Let π be the population proportion of the defective nails -

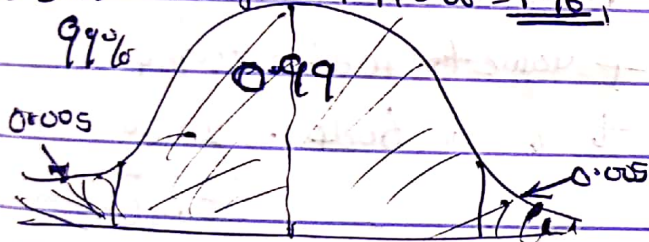
p = Sample proportion = $\frac{n}{N}$ of defective in the sample

n.

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

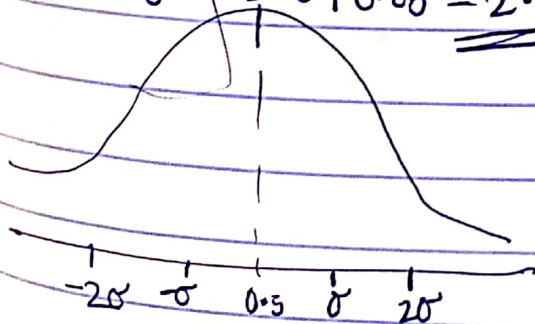


0.95 + 0.025 = 0.975. Check where 0.975 lies on z table that is under 0.06 which gives $1.9 + 0.06 = 1.96$



$$0.99 + 0.005 = 0.995$$

Z value of 0.995 = 2.58. 2.58 can be found 0.9951 = 0.995 can be found on z table under 0.08 = $2.5 + 0.08 = 2.58$



1

EXAMPLE

The variance of both height σ^2 produce in a industry is 100 while the mean height of 16 poles randomly selected from the industry is 185. Find 95% Confidence intervals for the population mean and mean.

Confidence Statement

Sol.

$$\sigma^2 = 100 \quad \text{i.e. } \sigma = 10$$

$$\bar{x} = 185, \quad n = 16$$

$$\bar{x} \pm Z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$185 \pm 1.96 \left(\frac{10}{\sqrt{16}} \right)$$

$$= 185 \pm 4.9$$

$$\text{Upper limit} = 185 + 4.9 = 189.9$$

$$\text{Lower limit} = 185 - 4.9 = 180.1$$

Confidence Statement $\Rightarrow$ At 95% Confidence level (probability) the interval 180.10 and 189.9 contain the mean height of the bolt population.

EXAMPLE

To illustrate Sample Estimate

$$p = \frac{\text{Number of Success in the Sample}}{n}$$

$$p = \frac{537}{1013} = 0.530$$

Now we want to generate Confidence interval for the point estimate.

Firstly assume a Confidence level say 95%

$$\pi = p \pm Z^* \sqrt{\frac{p(1-p)}{n}}$$

$$(0.499, 0.561)$$

$$np \gg 10$$

CONFIDENCE INTERVAL FOR μ

When σ is unknown and usefull when the sample size is large.

$$\bar{x} = \mu, \quad \sigma_x = \frac{\sigma}{\sqrt{n}}$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

σ is unknown and the sample size is small, use t-table.

$$-1.96 < \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < 1.96$$

$$\bar{x} - 1.96 \left(\frac{\sigma}{\sqrt{n}} \right) < \mu < \bar{x} + 1.96 \left(\frac{\sigma}{\sqrt{n}} \right)$$

T DISTRIBUTION

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

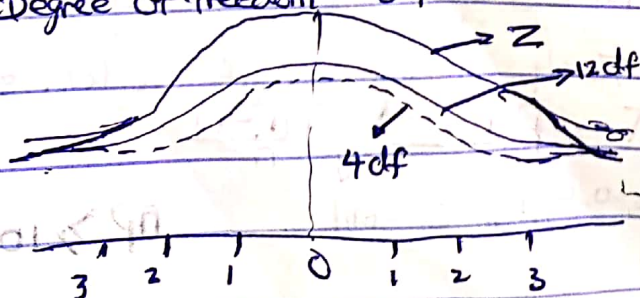
INTRODUCTION TO T-DISTRIBUTION

→ The spread of t-distribution is more than that of normal distribution.

→ As the no. of samples increases, your t-distribution tends towards normal distribution.

→ t-distribution are meant for small sample size only.

→ Degree of freedom $df = n - 1$



EXAMPLE

Here

→ Assuming df @ 99.5%

t-value → 4.32

→ From a sample of 49 students the mean mathematics score in a school was found to be 55% with a SD of 15. Find the 99% Confidence interval for the real mean score.

Sol

$$\bar{x} = 55\%, \quad n = 49, \quad s = 15$$

$$\alpha = 99\%$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

If ur table is not having the df

t value for 40df = 2.704

t " " 60df = 2.660

5.364

Through Interpolation and extrapolation

$$\frac{5.364}{2} = 2.682$$

$$\approx 2.7$$

PG 7

Inferential Statistics helps us to draw conclusion beyond the data we have from the population.

Statistical Inferences is the process of drawing conclusion about population parameters based on a sample taken from the population.

Sample $\Rightarrow$ This is the data collected from population.

TYPES OF STATISTICAL ANALYSIS

- Descriptive Statistics.
- Inferential Statistics.

STATISTICAL INFERENCE 2nd Pdf

Sampling distribution of $\bar{x}$ provides formula about the long run behaviour of when sample after sample is selected.

Sample proportion is the fraction of individual or objects in a sample that have some traits of interest or the population of individual a sample that possesses a particular property.

SAMPLING DISTRIBUTION OF MEAN

Rule 1 $\Rightarrow \mu_{\bar{x}} = \mu$

Rule 2 $\Rightarrow \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

For detail PG 2

Go to class on 9-6-2021

UNBIASED ESTIMATE $\Rightarrow$ This is a statistics whose mean values equal to the value of the population characteristics being estimated.

The Green letter π denotes the population of success in a population.

$p = \frac{\text{no of Success in the Sample}}{n}$

Statistics Population

$$\Rightarrow \mu_p = \pi$$

$\therefore p$ is an unbiased statistic for estimating π .

$\Rightarrow$ Standard deviation of p is

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

As long as n is large, $n\pi \geq 10$

and $n(1-\pi) \geq 10$. Also Sample Size is less than 10% of the population size.

CONFIDENCE INTERVAL 3rd Pdf

It is explained betw lower and upper endpoint of the interval.

$$p = \frac{\text{no of the Sample that possess the Property of Interest}}{n}$$

when n is large,

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

CONFIDENCE INTERVAL FOR POPULATION MEAN (μ)

prop of sampling distribution

$$\Rightarrow \mu_{\bar{x}} = \mu$$

$$\Rightarrow \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$\Rightarrow$ As long as n is large ($n > 30$), the sampling distribution of $\bar{x}$ is approximately normal even when a population distribution itself is not normal.

Formular for Confidence Interval for a population mean

$\Rightarrow \bar{x}$ is Sample mean from random sample

$\Rightarrow$ The Sample size n is large ($n > 30$)

$\Rightarrow$ Population standard deviation is known

$$\bar{x} \pm (Z_{\text{critical value}}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

CONDITION NECESSARY TO CONSIDER B4 USING CONFERENCE INTERVAL (CI)

$$\bar{x} \pm Z_{\text{critical value}} \left(\frac{\sigma}{\sqrt{n}} \right)$$

1) σ must be known (Population standard deviation must be known)

2) Sample Size n must be large $n \geq 30$

Then use this formula for CI

$$\bar{x} \pm \left(Z_{\text{critical value}} \right) \times \left(\frac{\sigma}{\sqrt{n}} \right)$$

NOTE the following;

Recall that for a normal population distribution the sampling distribution ~~is normal~~ of $\bar{x}$ is normal even for small sample sizes.

∴ If n is small but population distribution is normal, same confidence interval formula just introduced can still be used

i.e. if n is small ($n < 30$) and it is reasonable to believe that distribution of values in the population is normal, a confidence interval for μ (when σ is known) is

$$\bar{x} \pm \left(Z_{\text{critical value}} \right) \left(\frac{\sigma}{\sqrt{n}} \right)$$

CONFIDENCE INTERVAL FOR μ WHEN σ IS UNKNOWN

$$\mu_{sc} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

The above formula is used if σ is unknown and n is large ($n \geq 30$)

T - DISTRIBUTION

This t -distribution is used only when σ is unknown and n is small ($n < 30$)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

POINT ESTIMATION

CONFIDENCE INTERVAL FOR π

Population Proportion

$$\hat{p} = \frac{\text{no of successes in the sample}}{n}$$

PG 8

$$\Rightarrow \mu_p = \pi$$

This is used when n is large

$$\Rightarrow \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

$\Rightarrow$ As long as n ($n\pi \geq 10$) and

$n(1-\pi) \geq 10$ and sample size is less than 10% of the population size.

$$p \pm \left(Z_{\text{critical value}} \right) \times \left(\sqrt{\frac{\pi(1-\pi)}{n}} \right)$$

If π is unknown, as long as

sample size is large i.e. n is large

we can use $\sqrt{\frac{p(1-p)}{n}}$ in place of $\sqrt{\frac{\pi(1-\pi)}{n}}$

$$i.e. p \pm \left(Z_{\text{critical value}} \right) \left(\sqrt{\frac{p(1-p)}{n}} \right)$$

i.e. the following are to be noted;

$\Rightarrow np \geq 10$ and $n(1-p) \geq 10$

$\Rightarrow$ Sample size n is less than 10% of the population size if sampling is without replacement

$\Rightarrow$ The sample can be regarded as a random sample from the population of interest.

∴ If $np \geq 10$ and $n(1-p) \geq 10$

Then use $p \pm Z_{\text{critical value}} \left(\sqrt{\frac{p(1-p)}{n}} \right)$

- Online
- ① $np > 5$ ② $n(1-p) > 5$ ^{Both minutes}
 - ① $np > 10$ ② $n(1-p) > 10$ ¹⁹⁹⁰⁻²⁰⁰⁰
 - ① $np > 15$ ② $n(1-p) > 15$ ^{Current}

If the above condition is met, the sampling distribution of sample proportion is approximately normal (i.e.) mean $\mu = p$.

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Where $\hat{p} = \frac{x}{n}$
 $p = \text{Success}$

NOTE GENERALLY:

SAMPLING DISTRIBUTION (SD)

S.D for mean $= \bar{x} \pm Z_{\alpha} \frac{\sigma}{\sqrt{n}}$
 Note $\sigma_x = \frac{\sigma}{\sqrt{n}}$

The above formula is used the mention confidence interval, confidence limit, probability or if they mention mean in the question.

Note for Confidence interval = (lower, upper)
 Confidence limit = (Value $\pm$ Value)

SAMPLING DISTRIBUTION FOR STANDARD DEVIATION

$$S \pm Z_{\alpha} \left(\frac{s}{\sqrt{n}} \right)$$

Where S = Standard deviation of sample
 Example is ASS with electric bulb.
 The above formula is used when $n > 30$.

Sampling distribution for variance
 " " " median
 the above is not considered for now.

NORMAL DISTRIBUTION

Under normal distribution,

To find the final answer,
 If u are using 2-tail Z-table, u will subtract that value from ~~0.5~~ ¹ which will give u the exact final answer.
 If u are using 1-tail Z-table, u subtract that Z value being gotten from 0.5 which will give u the exact final value.

IDENTIFYING 1 tail or 2 tail Z-table

For 2-tail Z-table, first row first column u will see 0.5000.
 For 1-tail Z-table, first row first column, u will see 0.0000.

If $n < 30$ and σ is known, u can also use the above formula.

NOTE Under population proportion using;

$$p \pm Z_{\alpha} \sqrt{\frac{p(1-p)}{n}}$$

$np > 10$ and also $n\pi > 10$
 $n(1-p)$ or $n(1-\pi) > 10$

15 Curren

met the sample proportion mean $\mu = p$.

Also it is used if $n \geq 30$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

any where u see dat Cal any Percentage (99%, 90% etc) lesser, greater or both something, know dat is normal distribution.

$\bar{x}$ = Sample mean.

μ = Population mean.

LLY.

UTION (SD)

$$Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Example is ASS with 224 of water last 6 leaves, ie ~~6~~ last 6 pages first column and second column.

Here, u must know the type of z table u use either 2-tail (subtract what u get from z table from 1) and if it is 1-tail (u subtract the z value from 0.5)

NOTE At 2-tail z-table, ~~row~~ first column, use 0.5000

CONFIDENCE INTERVAL (CI)

Generally for CI, we use

$$\mu = \bar{x} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \text{ as the formula.}$$

It also applies when $n \geq 30$.

If $n < 30$ and σ is known, u can also use the above formula.

STANDARD

NOTE Under population proportion using;

$$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$

$np \geq 10$ and also $nq \geq 10$

of sample

But in d qn, if u see
proportion in SD, CI or Confidence
limit, use.

$$\pi = p \pm Z^* \sqrt{\frac{p(1-p)}{n}} \rightarrow \text{success}$$

where p = Sample proportion.

Given by $p = \frac{\text{no of success in the sample}}{n}$

~~$p = \frac{\text{no of success}}{n}$~~

Note $Z^* = Z_{\frac{\alpha}{2}}$

also $q = 1 - p \Rightarrow$ failure.

we can rewrite the formula that

$$\pi = p \pm Z^* \sqrt{\frac{pq}{n}}$$

or

$$\pi = p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$$

Here also, $n \geq 30$.

U Can also use $\sqrt{\frac{\pi(1-\pi)}{n}}$ in place of $\sqrt{\frac{pq}{n}}$, which gives;

$$\pi = p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\pi(1-\pi)}{n}}$$

NOTE: if $n < 30$ and σ is unknown, U use T-distribution table which leads us to T-distribution.

T-DISTRIBUTION

$n < 30$, $\sigma =$ unknown $\Rightarrow$ Population standard deviation
 $\mu = \bar{x} \pm T^* \frac{s}{\sqrt{n}}$ for CI
 i.e. $\bar{x} \pm T_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

But on normal t-distribution,

formula is $T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{\bar{x} - \mu}{\sigma_x}$

i.e. $T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ Which can be

used in Normal distribution but

only used when $n < 30$ and $\sigma =$ unknown.

Note;

Standard deviation is also known or called standard error.

i.e. Standard deviation = Standard Error

YouTube video.

NOTE

28-7-2021 class

$\Rightarrow$ All examples and assignment.

$\Rightarrow$ Note when to use lower T, upper T and middle T-table.

$\Rightarrow$ know when to use T-table and Z-table.

HYPOTHESIS

Null hypothesis is d one I have dat

U wanted to test.

Alternative is when it is opposite i.e. d statement is not true or less or greater than the sample.

And they are all Population mean μ .

Population mean

If σ is unknown

U use T-distribution $T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

but if σ is known, use

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\Rightarrow$ The breaking strength of cable has the mean $\mu = 1800$ lb and standard deviation $\sigma = 100$ lb by a new technique of producing cable and a sample of 50 cables is

$\bar{x} = 1850$ lb. Can we support this distribution in sampling distrib?

Sol

$\mu = 1800$ lb, $\sigma = 100$ lb

$n = 50$, $\bar{x} = 1850$

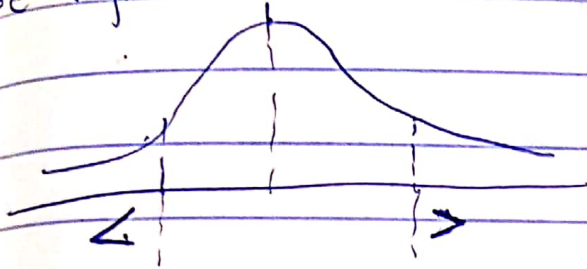
New Producing a new breaking strength

$$H_0: \mu = 1800$$

$$H_a: \mu > 1800$$

Null hypothesis

If H_a is greater, use 1 tail
the right tail. See below.



Significant level = 1 - Confidence level. Correct.

Test Statistics $\Rightarrow$ Z table

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}} = 3.54$$

from d gestion, 99% is $\alpha = 0.01$

$$= 0.9998$$

$$= 1 - 0.9998 = 0.0002$$

Standards are 99.70, 99% 99.73%

Find while p-value of 99% in
1 tail test is 2.33 instead of 2.58
in 2 tail.

Using One tail 99%

$$= 2.33 = H_a$$

Since: $H_a < 3.54$, the
test is wrong.

$\therefore H_0$ is not correct

$$3.54 - 3.5 = 0.04$$

$$1 - 0.04 = 0.96$$

$$0.96 = 96\%$$

If P value H_a

Note:

Since P value (2.33) $\times$ Z score 3.54
 $\therefore H_a$ is correct and H_0 is
not correct.

But if P value $>$ Z score,
 $\therefore H_a$ is not correct and H_0 is
Correct.